

GLOBAL OPTIMIZATION

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Fall 2018

These slides are motivated from Prof. Alex Aiken and Prof. Calvin Lin



Other Global Optimization:

- Constant Propagation
- Dead-code elimination
- Liveness analysis
- Common subexpression elimination
- Loop optimization

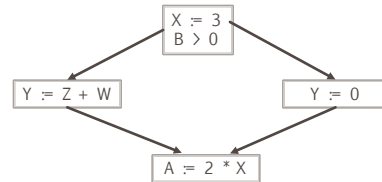
Local Optimization

- Recall the simple basic-block optimizations
 - Constant propagation
 - Dead code elimination

$X := 3$
 $Y := Z * W$
 $Q := X + Y$
 \longrightarrow
 $X := 3$
 $Y := Z * W$
 $Q := 3 + Y$
 \longrightarrow
 $Y := Z * W$
 $Q := 3 + Y$

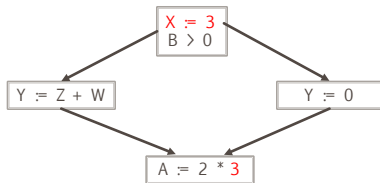
Global Optimization

- These optimizations can be extended to an entire control-flow graph



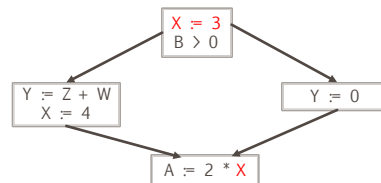
Global Optimization

- These optimizations can be extended to an entire control-flow graph



Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:



Correctness (cont.)

To replace a use of x by a constant k we must know that:

On every path to the use of x , the last assignment to x is

$$x := k^{**}$$

- The correctness condition is not trivial to check
- "All paths" includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
 - An analysis of the entire control-flow graph

Global Analysis

- Global optimization tasks share several traits:
 - The optimization depends on knowing a property X at a particular point in program execution
 - Proving X at any point requires knowledge of the entire program
 - It is OK to be conservative. If the optimization requires X to be true, then want to know either
 - X is definitely true
 - Don't know if X is true
 - It is always safe to say "don't know"

Global Analysis (cont.)

- Global dataflow analysis is a standard technique for solving problems with these characteristics
- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

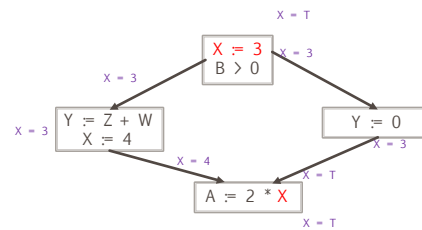
- Global constant propagation can be performed at any point where $**$ holds
- Consider the case of computing $**$ for a single variable X at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	interpretation
\perp	This statement never executes
c	$X = \text{constant } c$
T	X is not a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
 - Simply inspect the $x = ?$ associated with a statement using x
 - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties $x = ?$

Using the Information

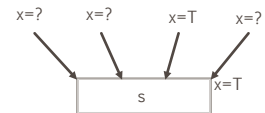
- The idea is to “push” or “transfer” information from one statement to the next
- For each statement s , we compute information about the value of x immediately before and after s

$C(s,x,in)$ = value of x before s
 $C(s,x,out)$ = value of x after s

Transfer Functions

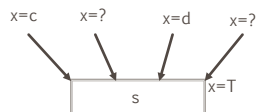
- Define a transfer function that transfers information one statement to another
- In the following rules, let statement s have immediate predecessor statements p_1, \dots, p_n

Rule 1



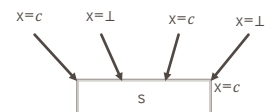
if $C(p_i, x, out) = T$ for any i , then $C(s, x, in) = T$

Rule 2



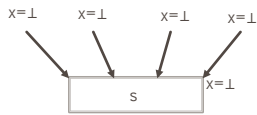
$C(p_1, x, out) = c$ & $C(p_i, x, out) = d$ & $d < c$
 then $C(s, x, in) = T$

Rule 3



if $C(p_i, x, out) = c$ or \perp for all i ,
 then $C(s, x, in) = c$

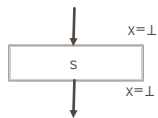
Rule 4



if $C(p_i, x, out) = \perp$ for all i ,
then $C(s, x, in) = \perp$

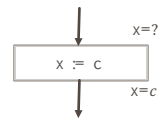
-
- Rules 1-4 relate the out of one statement to the in of the next statement
 - Now we need rules relating the in of a statement to the out of the same statement

Rule 5



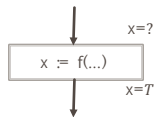
$C(s, x, out) = \perp$
if $C(s, x, in) = \perp$

Rule 6



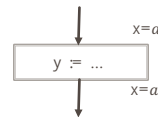
$C(x := c, x, out) = c$ if c is a constant

Rule 7



$C(x := f(...), x, out) = T$

Rule 8



$C(y := ..., x, out) = C(y := ..., x, in)$ if $x <> y$

Algorithm

1. For every entry s to the program, set $C(s, x, in) = T$
 2. Set $C(s, x, in) = C(s, x, out) = \perp$ everywhere else
 3. Repeat until all points satisfy 1-8:
 - Pick s not satisfying 1-8 and update using the appropriate rule
- **Ordering:**
 - We can simplify the presentation of the analysis by ordering the values $\perp < c < T$

Common subexpression elimination

- Example:

$a := b + c$	\Rightarrow	$a := b + c$
$c := b + c$		$c := a$
$d := b + c$		$d := b + c$
- Example in array index calculations
 - $c[i+1] := a[i+1] + b[i+1]$
 - During address computation, $i+1$ should be reused
 - Not visible in high level code, but in intermediate code

Code Elimination

- **Unreachable code elimination**
 - Construct the control flow graph
 - Unreachable code block will not have an incoming edge
 - After constant propagation/folding, unreachable branches can be eliminated
- **Dead code elimination**
 - Ineffective statements (immediately redefined, eliminated)

$x := y + 1$	\Rightarrow	$y := 5$
$y := 5$		$x := 2 * z$
$x := 2 * z$		$x := 2 * z$
 - A variable is dead if it is never used after last definition
 - Eliminate assignments to dead variables
- Need to do data flow analysis to find dead variables

Function Optimization

- **Function inlining**
 - Replace a function call with the body of the function
 - Save a lot of copying of the parameters, return address, etc.
- **Function cloning**
 - Create specialized code for a function for different calling parameters

Loop Optimization

- **Loop optimization**
 - Consumes 90% of the execution time
 - \Rightarrow a larger payoff to optimize the code within a loop
- **Techniques**
 - Loop invariant detection and code motion
 - Induction variable elimination
 - Strength reduction in loops
 - Loop unrolling
 - Loop peeling
 - Loop fusion

Loop Optimization

- **Loop invariant detection**
 - If the result of a statement or expression does not change within a loop, and it has no external sideeffect
 - Computation can be moved to outside of the loop
 - Example


```
for (i=0; i<n; i++) a[i] := a[i] + x/y;
```

 - Three address code


```
for (i=0; i<n; i++) { c := x/y; a[i] := a[i] + c; }
               $\Rightarrow$  c := x/y;
              for (i=0; i<n; i++) a[i] := a[i] + c;
```

Loop Optimization

• Code Motion

- Reduce frequency with which computation performed
 - If it will always produce same result
 - Especially moving code out of loop

```

for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
    
```

→

```

for (i = 0; i < n; i++) {
  int ni = n*i;
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
}
    
```

Loop Optimization

• Strength reduction in loops

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide
 - 16*x → x << 4
 - Depends on cost of multiply or divide instruction
- Recognize sequence of products

```

for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
    
```

→

```

int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
    
```

Loop Optimization

• Strength reduction in loops

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide
 - 16*x → x << 4
 - Depends on cost of multiply or divide instruction
- Recognize sequence of products

```

s := 0;
for (i=0; i<n; i++)
{
  v := 4 * i;
  s := s + v;
}
    
```

→

```

s := 0;
for (i=0; i<n; i++)
{
  v := v + 4;
  s := s + v;
}
    
```

Loop Optimization

• Induction variable elimination

- If there are multiple induction variables in a loop, can eliminate the ones which are used only in the test condition
- Example
 - s := 0; for (i=0; i<n; i++) { s := 4 * i; ... } - i is not referenced in loop
 - ⇒ s := 0; e := 4*n; while (s < e) { s := s + 4; }

```

s := 0;
for (i=0; i<n; i++)
{ s := 4 * i; ... }
-- i is not referenced in loop
    
```

→

```

s := 0;
e := 4*n;
while (s < e) {
  s := s + 4;
}
    
```

Code Optimization Techniques

• Loop unrolling

- Execute loop body multiple times at each iteration
- Get rid of the conditional branches, if possible
- Allow optimization to cross multiple iterations of the loop
 - Especially for parallel instruction execution
- Space time tradeoff
 - Increase in code size, reduce some instructions

• Loop peeling

- Similar to unrolling
- But unroll the first and/or last few iterations

Loop Optimization

• Loop fusion

- Example

<pre> for i=1 to N do A[i] = B[i] + 1 endfor for i=1 to N do C[i] = A[i] / 2 endfor for i=1 to N do D[i] = 1 / C[i+1] endfor </pre>	<pre> for i=1 to N do A[i] = B[i] + 1 C[i] = A[i] / 2 D[i] = 1 / C[i+1] endfor </pre>
---	---

Before Loop Fusion

Loop Optimization

- Loop fusion

- Example

```

for i=1 to N do
  A[i] = B[i] + 1
endfor
for i=1 to N do
  C[i] = A[i] / 2
endfor
for i=1 to N do
  D[i] = 1 / C[i+1]
endfor

```

```

for i=1 to N do
  A[i] = B[i] + 1
  C[i] = A[i] / 2
  D[i] = 1 / C[i+1]
endfor

```

Is this correct?
Actually, cannot fuse
the third loop

Before Loop Fusion

Limitations of Compiler Optimization

- Operate Under Fundamental Constraint

- Must not cause any change in program behavior under any possible condition
- Often prevents it from making optimizations when would only affect behavior under pathological conditions.

- Behavior that may be obvious to the programmer can be obfuscated by languages and coding styles

- e.g., data ranges may be more limited than variable types suggest

- Most analysis is performed only within procedures

- whole-program analysis is too expensive in most cases

- Most analysis is based only on static information

- compiler has difficulty anticipating run-time inputs

- When in doubt, the compiler must be conservative