

GLOBAL OPTIMIZATION

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Other Global Optimization:

- Constant Propagation
- Dead-code elimination
- Liveness analysis
- Common subexpression elimination
- Loop optimization

Local Optimization

- Recall the simple basic-block optimizations
 - Constant propagation
 - Dead code elimination

X := 3
Y := Z * W
Q := X + Y



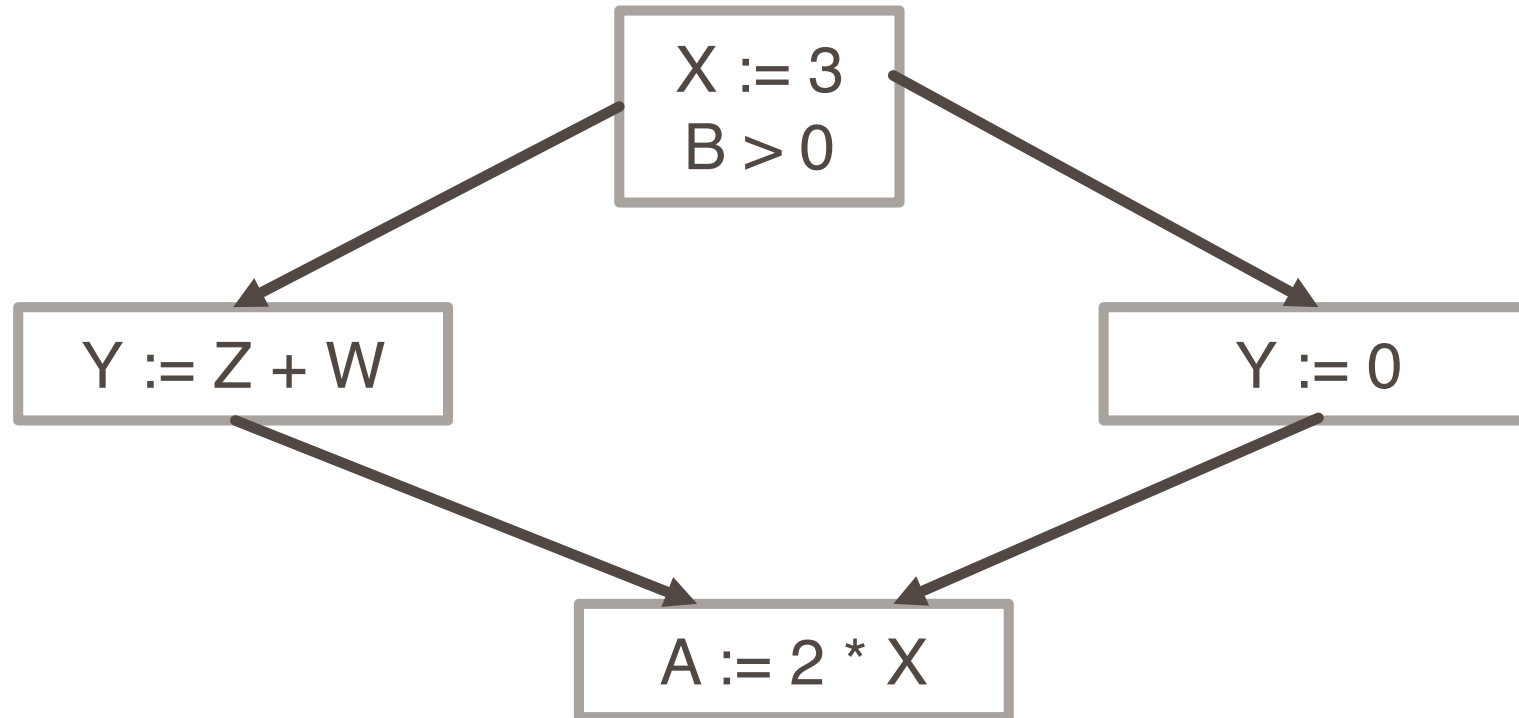
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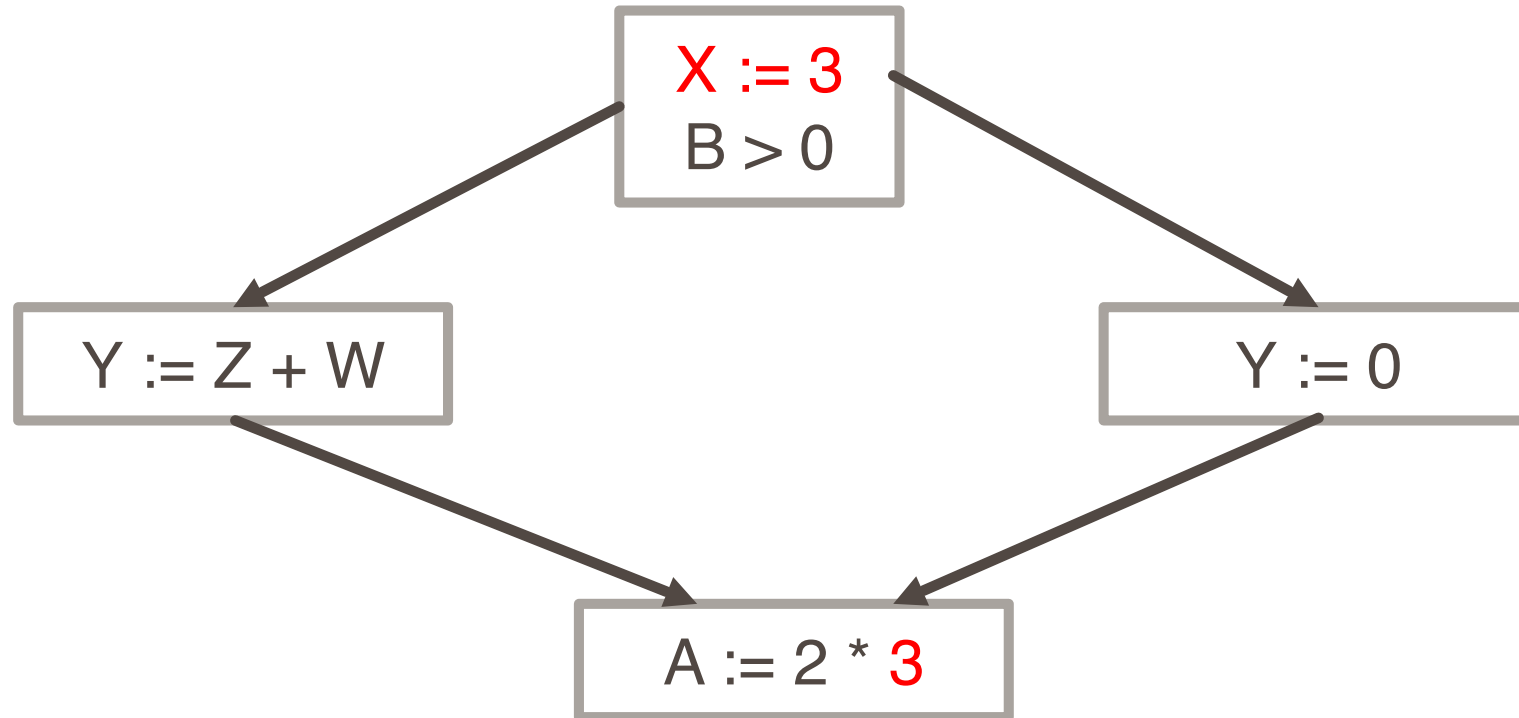
Global Optimization

- These optimizations can be extended to an entire control-flow graph



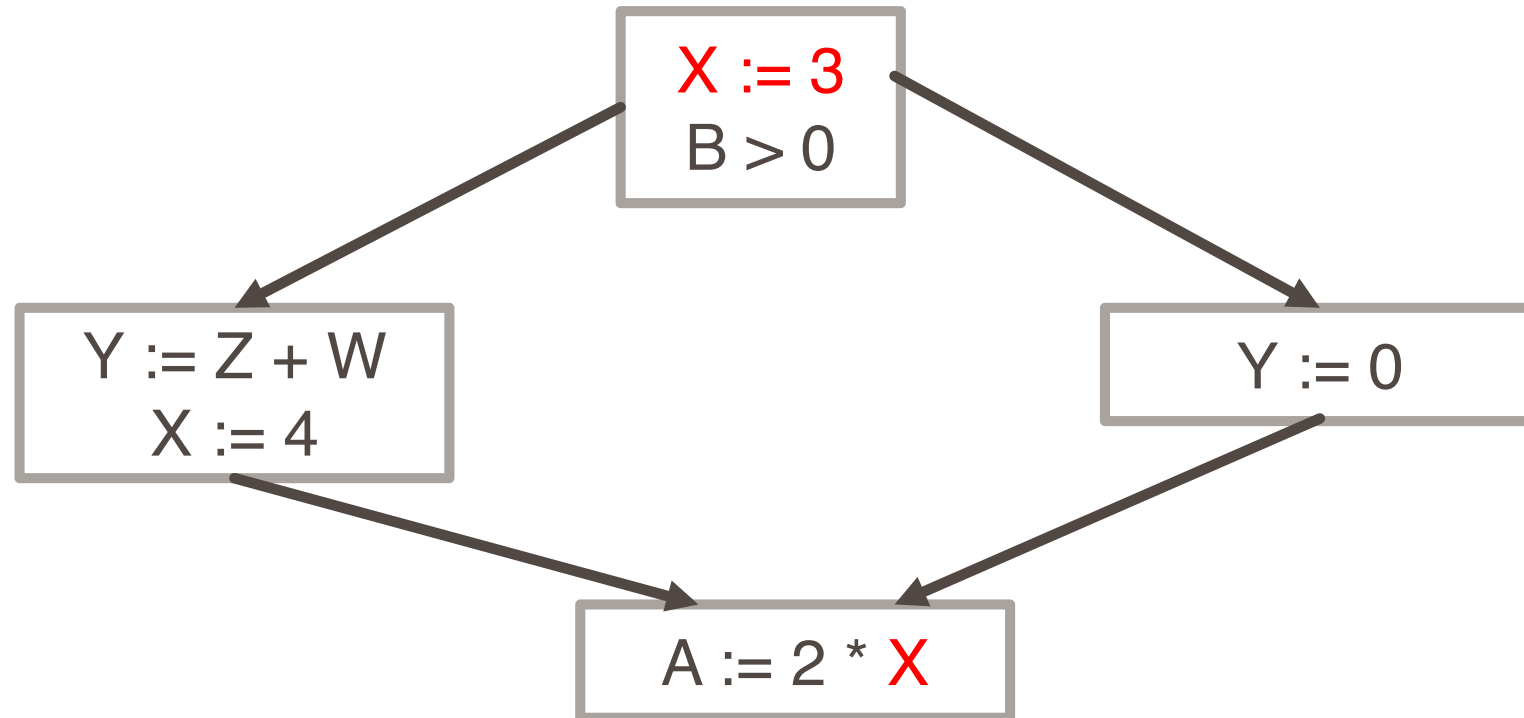
Global Optimization

- These optimizations can be extended to an entire control-flow graph



Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:



Correctness (cont..)

To replace a use of x by a constant k we must know that:

On every path to the use of x , the last assignment to x is

$x := k$

- The correctness condition is not trivial to check
- “All paths” includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
 - An analysis of the entire control-flow graph

Global Analysis

- Global optimization tasks share several traits:
 - The optimization depends on knowing a property X at a particular point in program execution
 - Proving X at any point requires knowledge of the entire program
 - It is OK to be conservative. If the optimization requires X to be true, then want to know either
 - X is definitely true
 - Don't know if X is true
 - It is always safe to say "don't know"

Global Analysis (cont..)

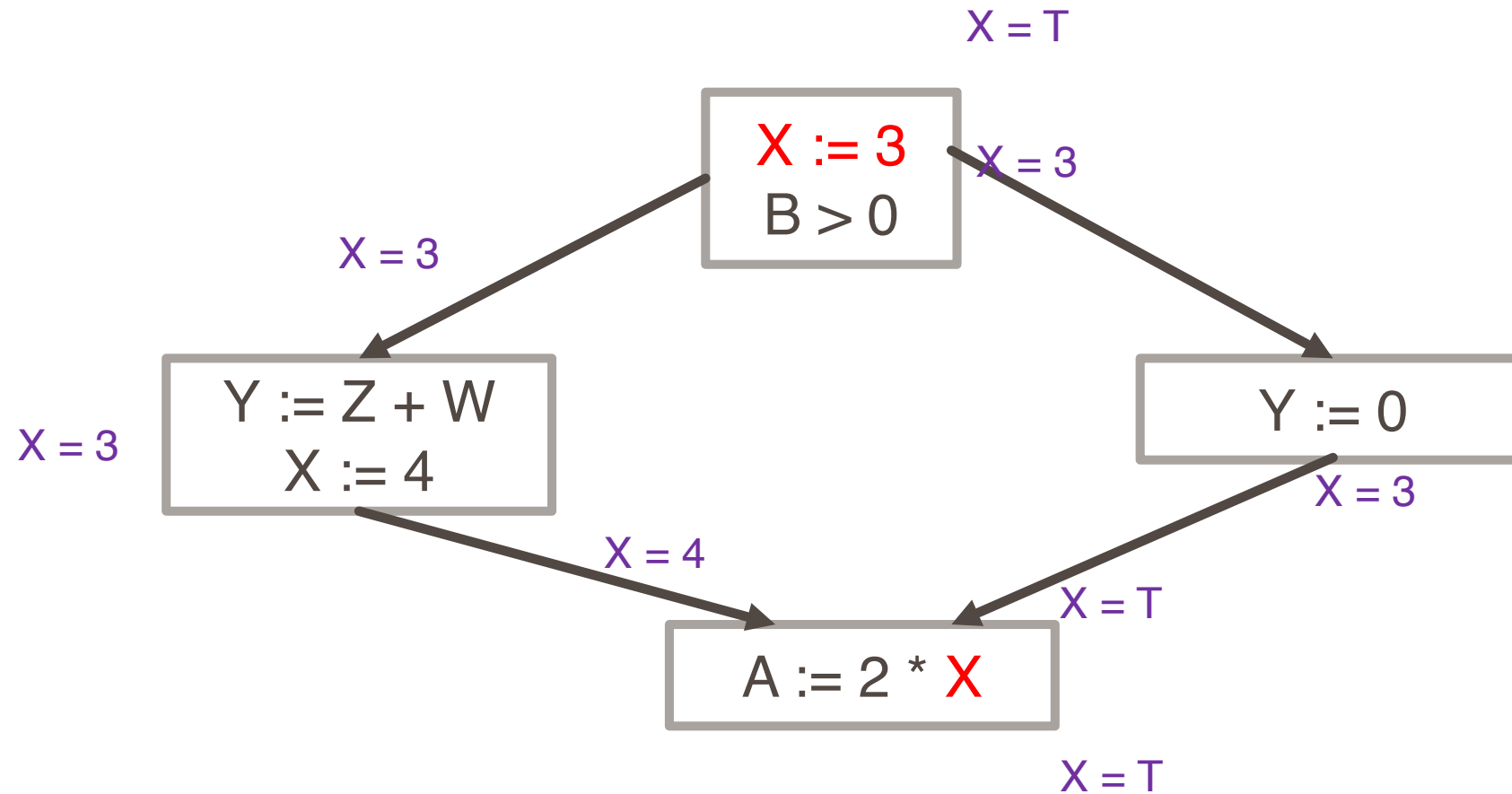
- Global dataflow analysis is a standard technique for solving problems with these characteristics
- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	interpretation
\perp (“bottom”)	This statement never executes
c	$X = \text{constant } c$
T (“top”)	X is not a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
 - Simply inspect the $x = ?$ associated with a statement using x
 - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties $x = ?$

Using the Information

- The idea is to “push” or “transfer” information from one statement to the next
- For each statement s , we compute information about the value of x immediately before and after s

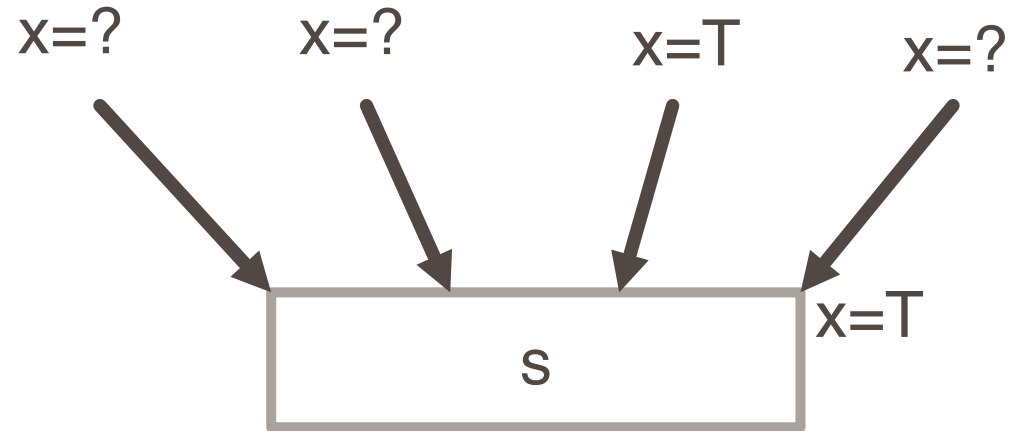
$C(s,x,in)$ = value of x before s

$C(s,x,out)$ = value of x after s

Transfer Functions

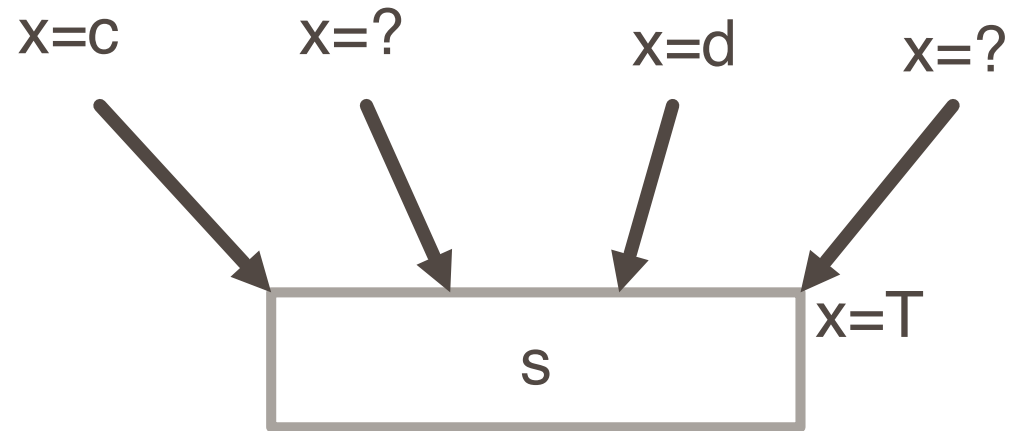
- Define a transfer function that transfers information one statement to another
- In the following rules, let statement s have immediate predecessor statements p_1, \dots, p_n

Rule 1



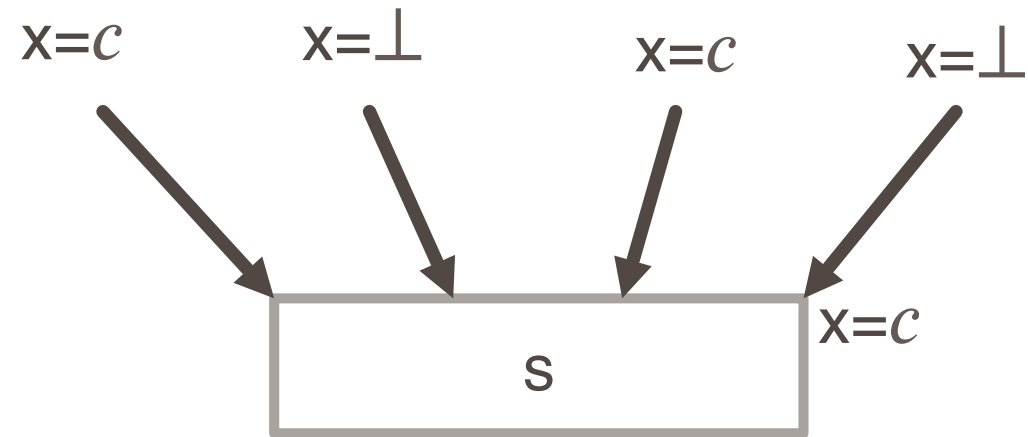
if $C(p_i, x, \text{out}) = T$ for any i , then $C(s, x, \text{in}) = T$

Rule 2



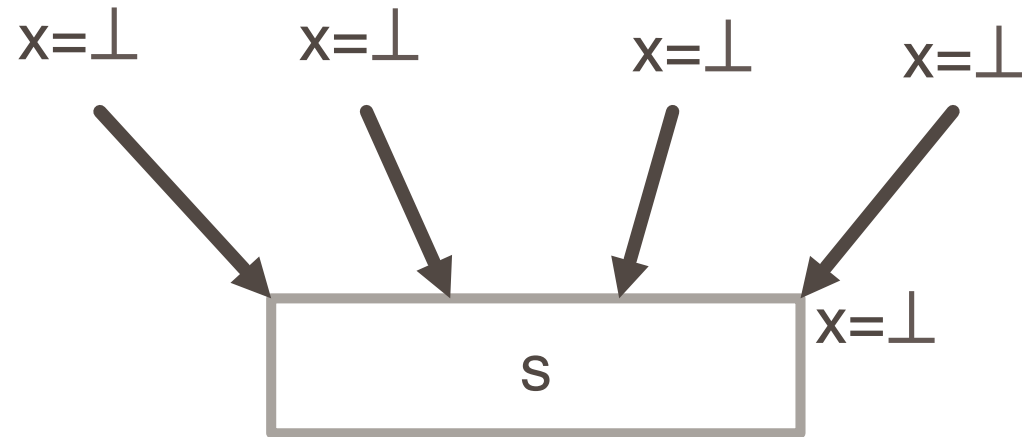
$C(p_i, x, \text{out}) = c \ \& \ C(p_j, x, \text{out}) = d \ \& \ d \triangleleft c$
then $C(s, x, \text{in}) = T$

Rule 3



if $C(p_i, x, \text{out}) = c$ or \perp for all i ,
then $C(s, x, \text{in}) = c$

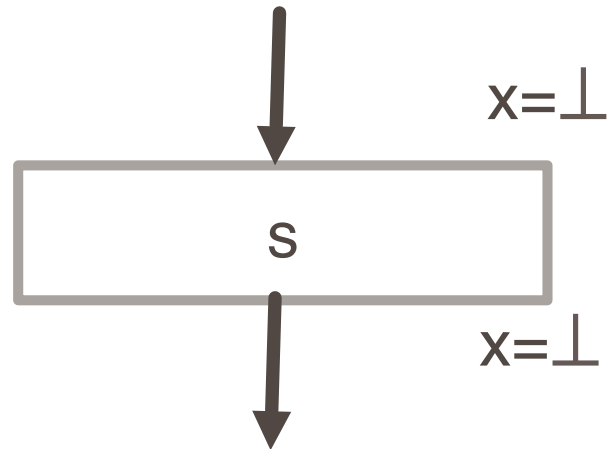
Rule 4



if $C(p_i, x, \text{out}) = \perp$ for all i ,
then $C(s, x, \text{in}) = \perp$

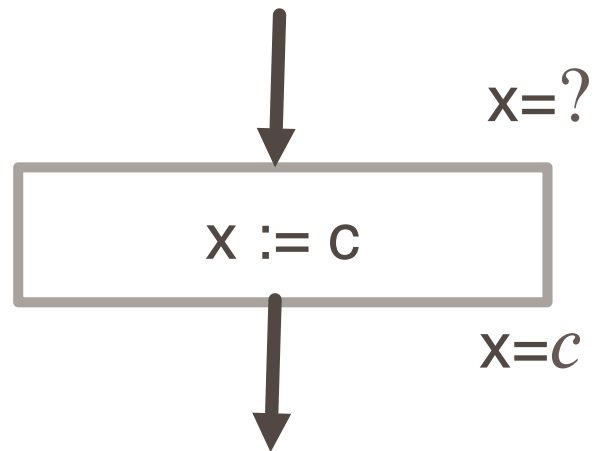
-
- Rules 1-4 relate the out of one statement to the in of the next statement
 - Now we need rules relating the in of a statement to the out of the same statement

Rule 5



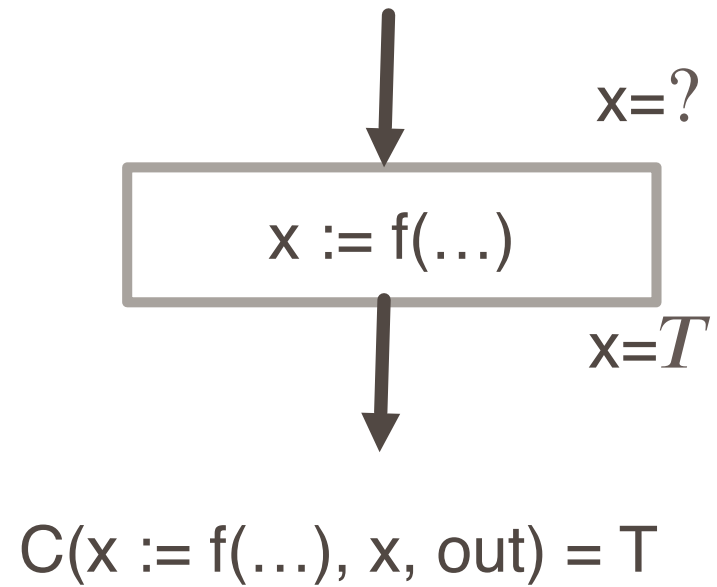
$$C(s, x, \text{out}) = \perp$$
$$\text{if } C(s, x, \text{in}) = \perp$$

Rule 6

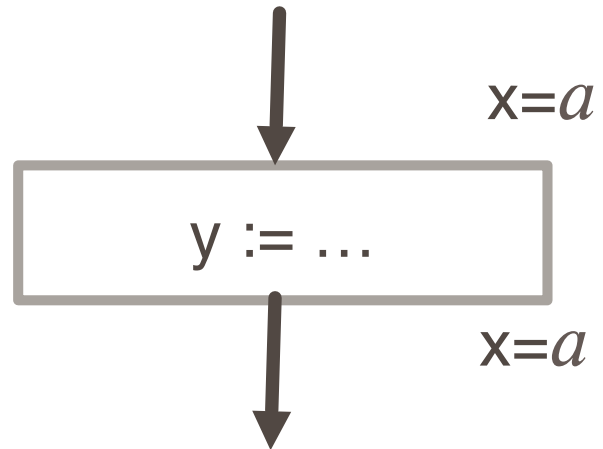


$C(x := c, x, out) = c$ if c is a constant

Rule 7



Rule 8



$$C(y := \dots, x, \text{out}) = C(y := \dots, x, \text{in}) \text{ if } x \diamond y$$

Algorithm

1. For every entry s to the program, set $C(s, x, \text{in}) = T$
 2. Set $C(s, x, \text{in}) = C(s, x, \text{out}) = \perp$ everywhere else
 3. Repeat until all points satisfy 1-2:
 - Pick s not satisfying 1-2 and update using the appropriate rule
-
- **Ordering:**
 - We can simplify the presentation of the analysis by ordering the values $\perp < c < T$

Common subexpression elimination

- Example:

a := b + c		a := b + c
c := b + c	⇒	c := a
d := b + c		d := b + c

- Example in array index calculations
 - $c[i+1] := a[i+1] + b[i+1]$
 - During address computation, $i+1$ should be reused
 - Not visible in high level code, but in intermediate code

Code Elimination

- **Unreachable code elimination**

- Construct the control flow graph
- Unreachable code block will not have an incoming edge
- After constant propagation/folding, unreachable branches can be eliminated

- **Dead code elimination**

- Ineffective statements

- $x := y + 1$ (immediately redefined, eliminate!)
- $y := 5$ \Rightarrow $y := 5$
- $x := 2 * z$ $x := 2 * z$

- A variable is dead if it is never used after last definition
 - Eliminate assignments to dead variables
- Need to do data flow analysis to find dead variables

Function Optimization

- **Function inlining**
 - Replace a function call with the body of the function
 - Save a lot of copying of the parameters, return address, etc.
- **Function cloning**
 - Create specialized code for a function for different calling parameters

Loop Optimization

- Loop optimization
 - Consumes 90% of the execution time
 - ⇒ a larger payoff to optimize the code within a loop
- Techniques
 - Loop invariant detection and code motion
 - Induction variable elimination
 - Strength reduction in loops
 - Loop unrolling
 - Loop peeling
 - Loop fusion

Loop Optimization

- **Loop invariant detection**

- If the result of a statement or expression does not change within a loop, and it has no external side-effect
- Computation can be moved to outside of the loop
- Example

```
for (i=0; i<n; i++) a[i] := a[i] + x/y;
```

- Three address code

```
for (i=0; i<n; i++) { c := x/y; a[i] := a[i] + c; }
```

⇒ c := x/y;

```
for (i=0; i<n; i++) a[i] := a[i] + c;
```

Loop Optimization

- **Code Motion**

- Reduce frequency with which computation performed
 - If it will always produce same result
 - Especially moving code out of loop

```
for (i = 0; i < n; i++)  
  for (j = 0; j < n; j++)  
    a[n*i + j] = b[j];
```



```
for (i = 0; i < n; i++) {  
  int ni = n*i;  
  for (j = 0; j < n; j++)  
    a[ni + j] = b[j];  
}
```

Loop Optimization

- Strength reduction in loops

- Replace costly operation with simpler one

- Shift, add instead of multiply or divide

$16 * x \quad \text{-->} \quad x \ll 4$

- Depends on cost of multiply or divide instruction

- Recognize sequence of products

```
for (i = 0; i < n; i++)  
  for (j = 0; j < n; j++)  
    a[n*i + j] = b[j];
```



```
int ni = 0;  
for (i = 0; i < n; i++) {  
  for (j = 0; j < n; j++)  
    a[ni + j] = b[j];  
  ni += n;  
}
```

Loop Optimization

- Strength reduction in loops

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide

$16 * x \quad \text{-->} \quad x \ll 4$

- Depends on cost of multiply or divide instruction
- Recognize sequence of products

```
s := 0;
for (i=0; i<n; i++)
{
    v := 4 * i;
    s := s + v;
}
```



```
s := 0;
for (i=0; i<n; i++)
{
    v := v + 4;
    s := s + v;
}
```


Loop Optimization

- Induction variable elimination

- If there are multiple induction variables in a loop, can eliminate the ones which are used only in the test condition

- Example

`s := 0; for (i=0; i<n; i++) { s := 4 * i; ... } -- i is not referenced in loop`

`⇒ s := 0; e := 4*n; while (s < e) { s := s + 4; }`

```
s := 0;
for (i=0; i<n; i++)
{ s := 4 * i; ... }
-- i is not referenced in
loop
```



```
s := 0;
e := 4*n;
while (s < e) {
  s := s + 4;
}
```

Code Optimization Techniques

- **Loop unrolling**
 - Execute loop body multiple times at each iteration
 - Get rid of the conditional branches, if possible
 - Allow optimization to cross multiple iterations of the loop
 - Especially for parallel instruction execution
 - Space time tradeoff
 - Increase in code size, reduce some instructions
- **Loop peeling**
 - Similar to unrolling
 - But unroll the first and/or last few iterations

Loop Optimization

- Loop fusion

- Example

```
for i=1 to N do
    A[i] = B[i] + 1
endfor
for i=1 to N do
    C[i] = A[i] / 2
endfor
for i=1 to N do
    D[i] = 1 / C[i+1]
endfor
```

```
for i=1 to N do
    A[i] = B[i] + 1
    C[i] = A[i] / 2
    D[i] = 1 /
        C[i+1]
endfor
```

Before Loop Fusion

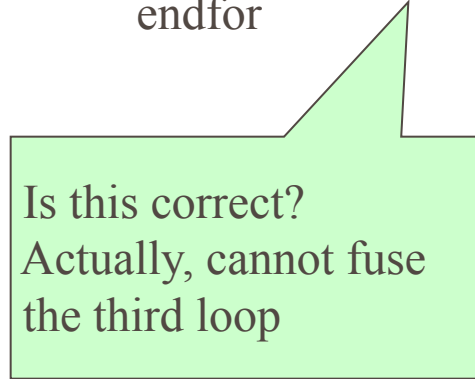
Loop Optimization

- Loop fusion

- Example

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for i=1 to N do
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endfor
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for i=1 to N do
    A[i] = B[i] + 1
    C[i] = A[i] / 2
    D[i] = 1 / C[i+1]
endfor
```



Is this correct?
Actually, cannot fuse
the third loop

Before Loop Fusion

Limitations of Compiler Optimization

- **Operate Under Fundamental Constraint**
 - Must not cause any change in program behavior under any possible condition
 - Often prevents it from making optimizations when would only affect behavior under pathological conditions.
- **Behavior that may be obvious to the programmer can be obfuscated by languages and coding styles**
 - e.g., data ranges may be more limited than variable types suggest
- **Most analysis is performed only within procedures**
 - whole-program analysis is too expensive in most cases
- **Most analysis is based only on static information**
 - compiler has difficulty anticipating run-time inputs
- **When in doubt, the compiler must be conservative**