# GLOBAL OPTIMIZATION

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These slides are motivated from Prof. Alex Aiken and Prof. Calvin Lin

# Other Global Optimization:

- Constant Propagation
- Dead-code elimination
- Liveness analysis
- Common subexpression elimination
- Loop optimization

# Local Optimization

- Recall the simple basic-block optimizations
  - Constant propagation
  - Dead code elimination

```
X := 3
Y := Z * W
Q := X + Y
```

→

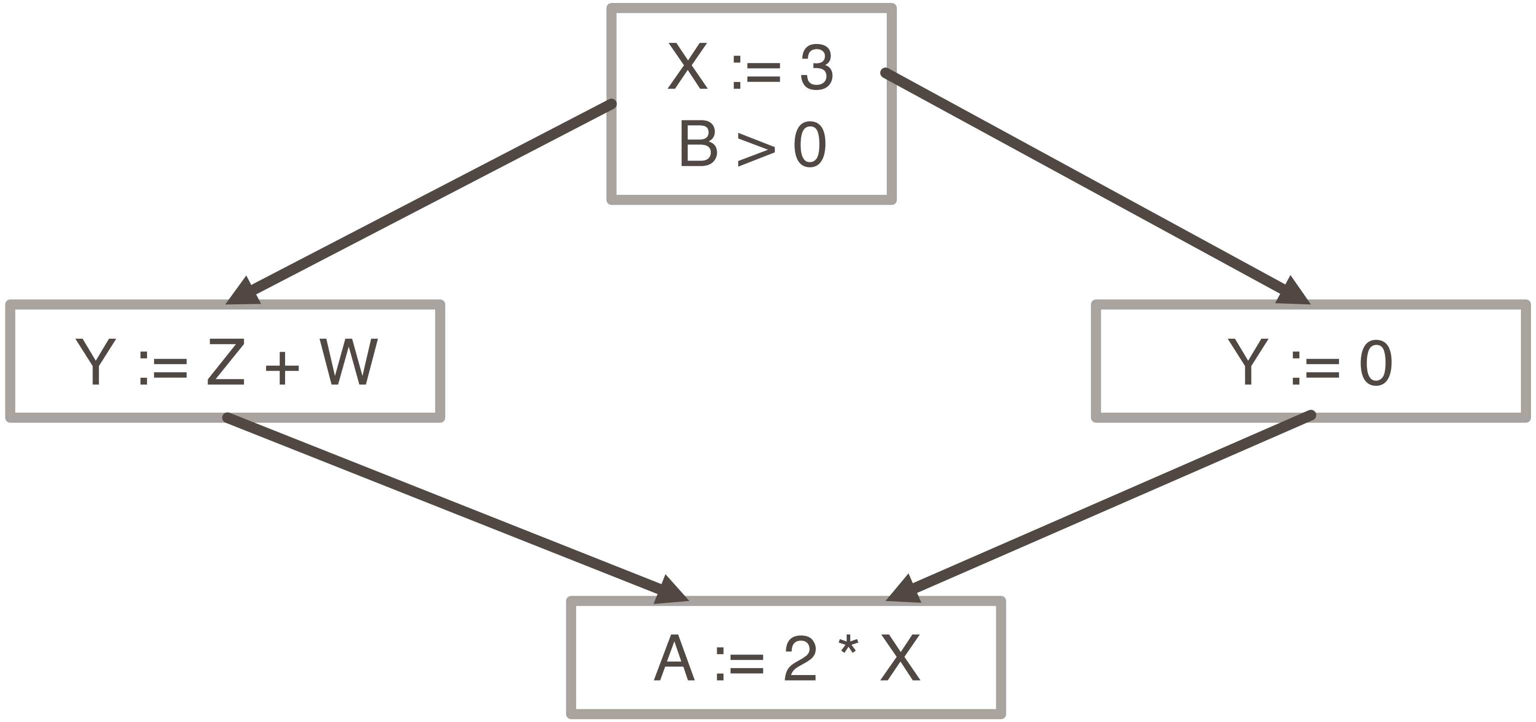
```
X := 3
Y := Z * W
Q := 3 + Y
```

→

```
Y := Z * W
Q := 3 + Y
```

# Global Optimization

- These optimizations can be extended to an entire control-flow graph



# Global Optimization

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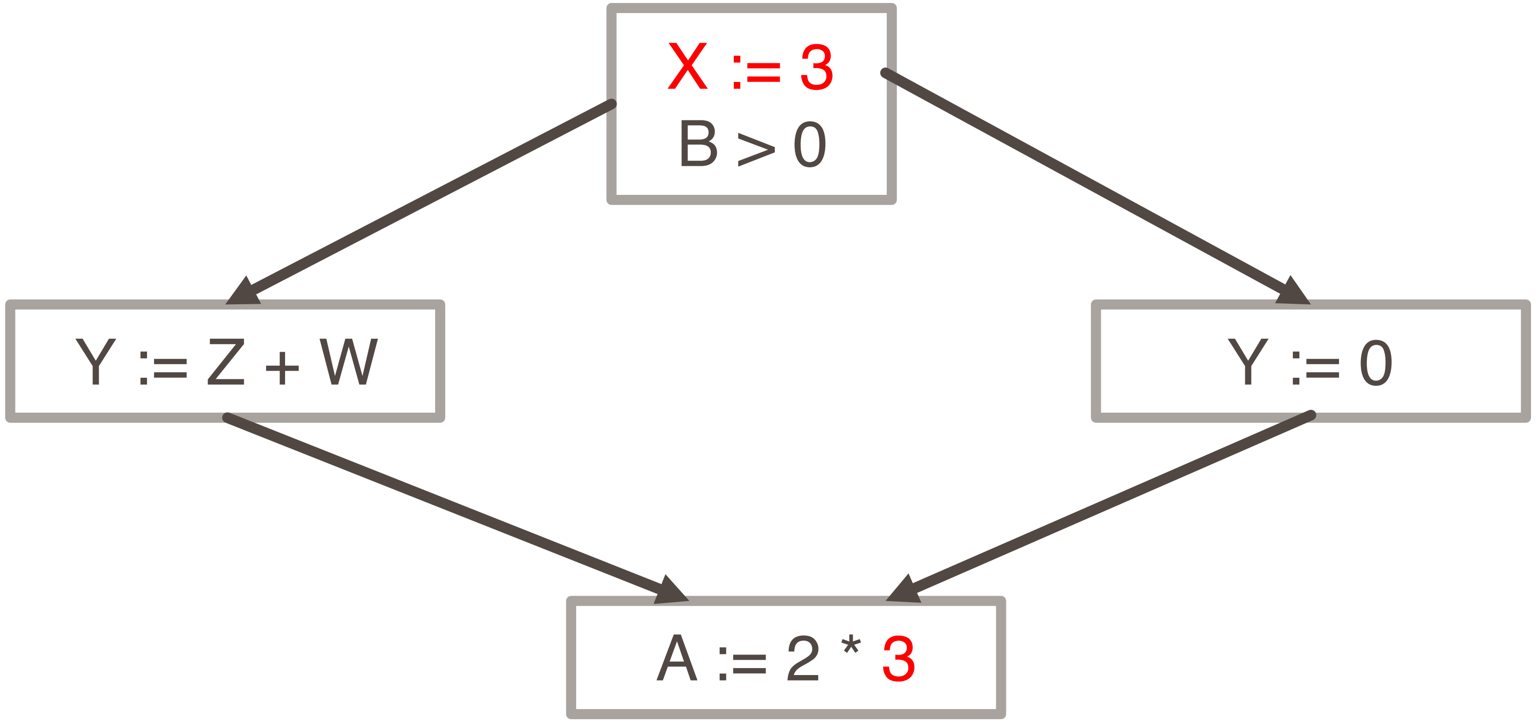
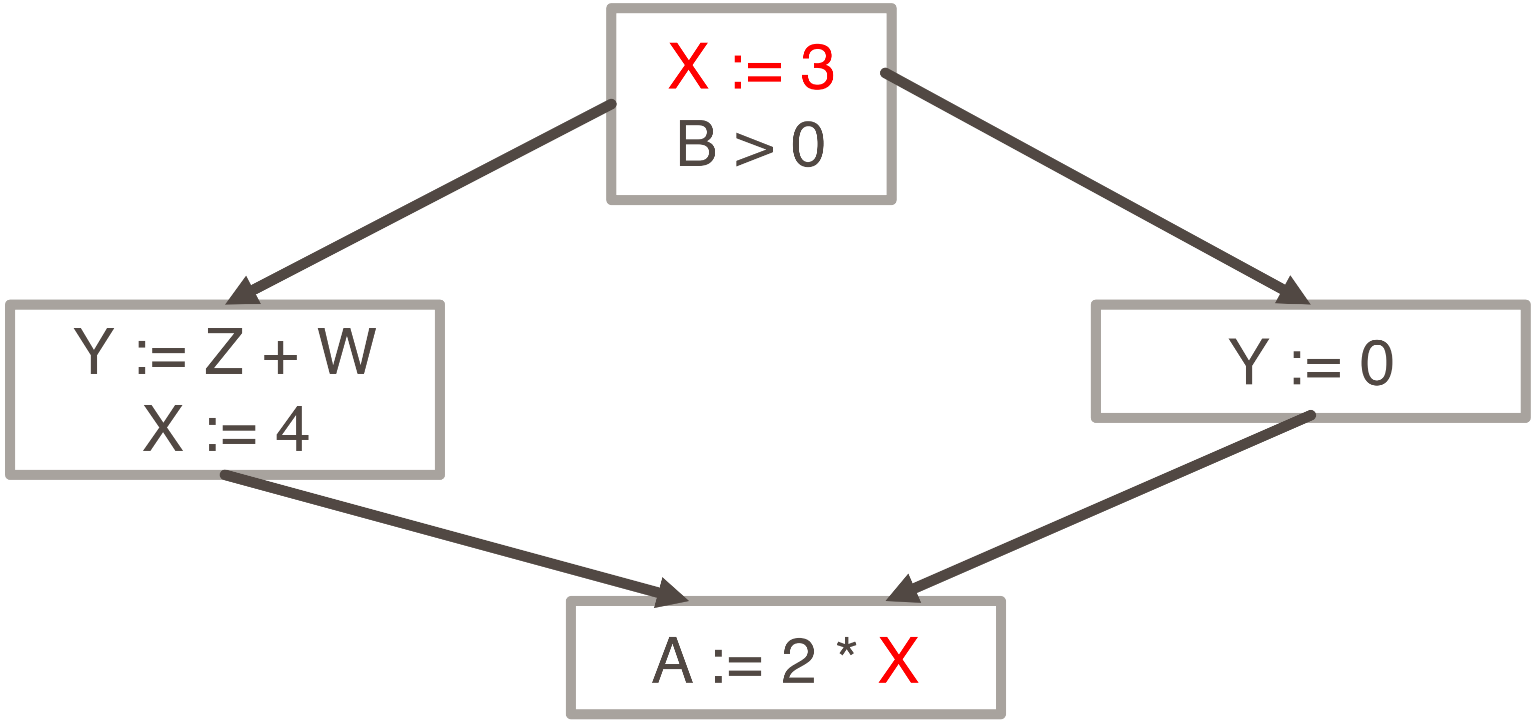


# Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:



# Correctness (cont..)

To replace a use of x by a constant k we must know that:

On every path to the use of x, the last assignment to x is

x := k **

- The correctness condition is not trivial to check
- “All paths” includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
    - An analysis of the entire control-flow graph

# Global Analysis

- Global optimization tasks share several traits:
  - The optimization depends on knowing a property X at a particular point in program execution
  - Proving X at any point requires knowledge of the entire program
  - It is OK to be conservative. If the optimization requires X to be true, then want to know either
    - X is definitely true
    - Don’t know if X is true
    - It is always safe to say “don’t know”

# Global Analysis (cont..)

- Global dataflow analysis is a standard technique for solving problems with these characteristics
- Global constant propagation is one example of an optimization that requires global dataflow analysis

# Global Constant Propagation

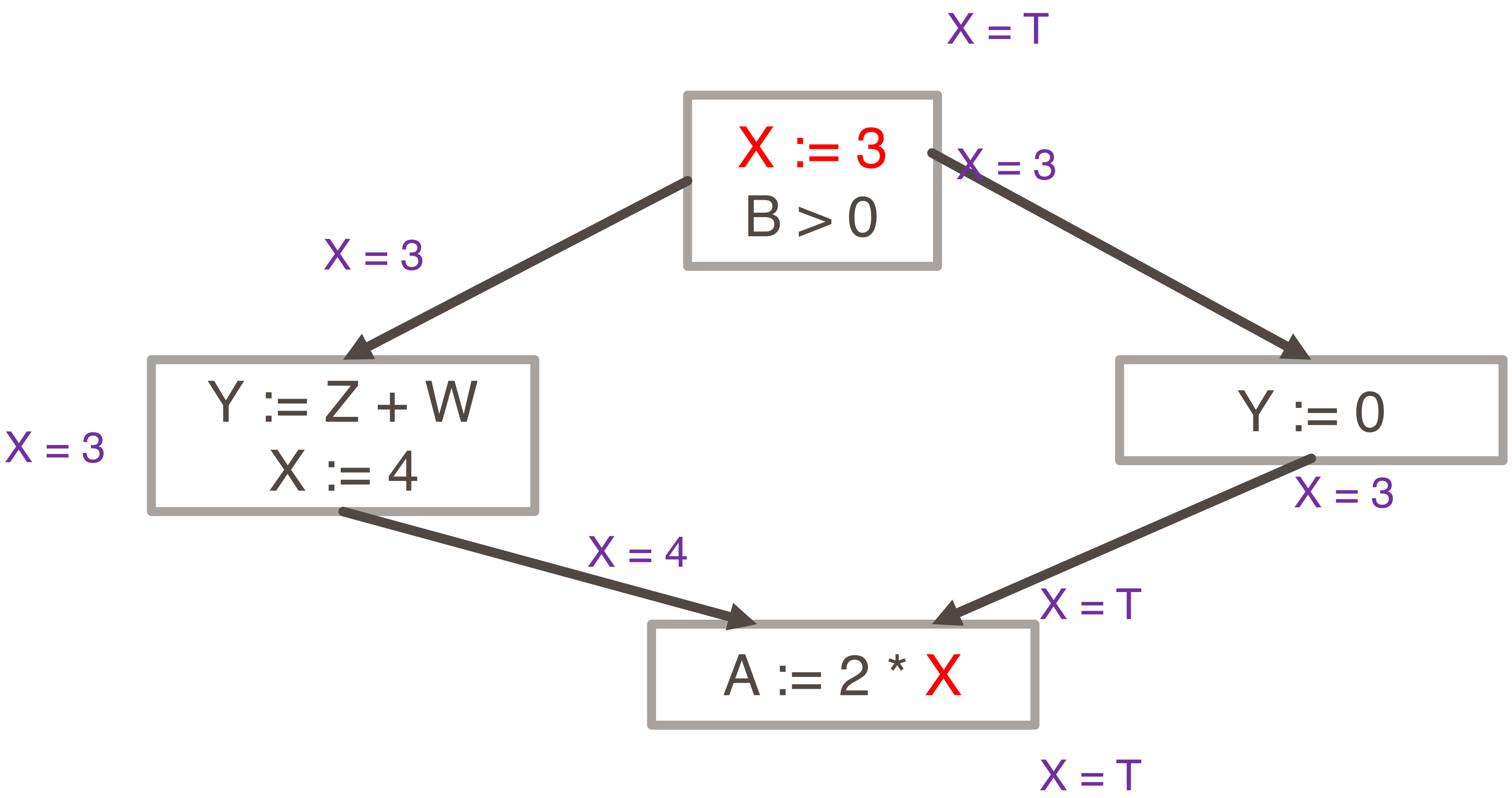
- Global constant propagation can be performed at any point where ** holds
- Consider the case of computing ** for a single variable X at all program points

# Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

| value | interpretation |
| --- | --- |
| $\perp$ ("bottom") | This statement never executes |
| c | X = constant c |
| T ("top") | X is not a constant |

# Example



# Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the x = ? associated with a statement using x
  - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties x = ?

# Using the Information

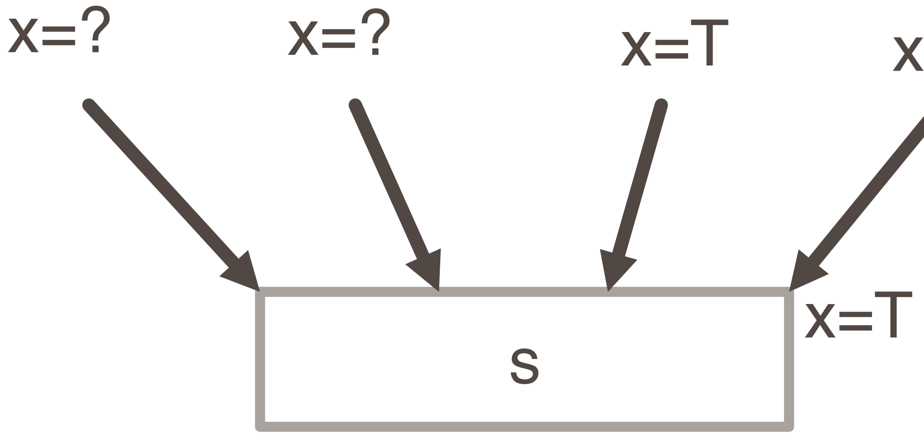
- The idea is to "push" or "transfer" information from one statement to the next
- For each statement s, we compute information about the value of x immediately before and after s

C(s,x,in) = value of x before s
C(s,x,out) = value of x after s

# Transfer Functions

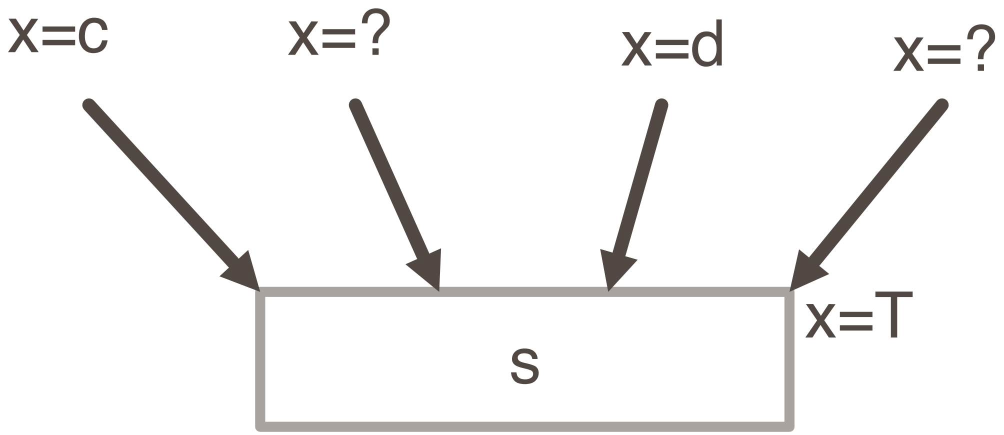
- Define a transfer function that transfers information one statement to another
- In the following rules, let statement $s$ have immediate predecessor statements $p_1,\ldots,p_n$

# Rule 1



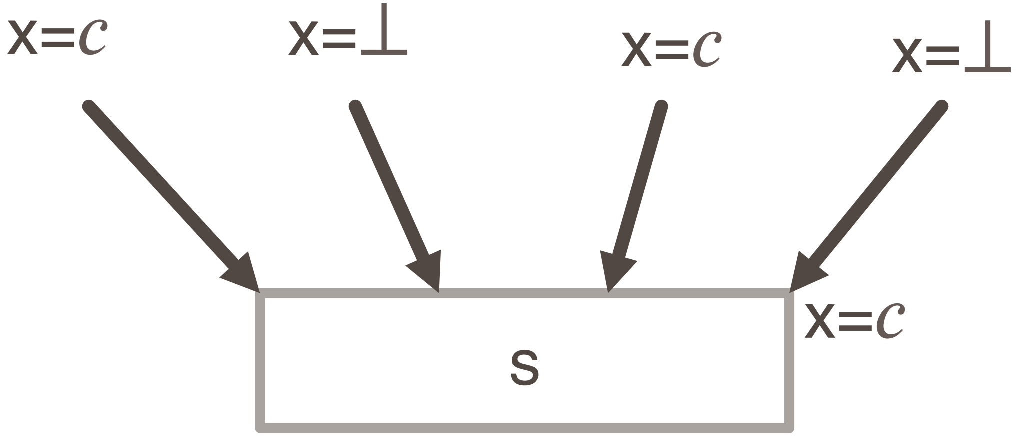

if $C(p_i , x, out) = T$ for any i, then $C(s, x, in) = T$

# Rule 2




$$C(p_i\ , x, out) = c \ \&\ C(p_j, x, out) = d \ \&\ d <> c$$
$$\text{then } C(s, x, in) = T$$

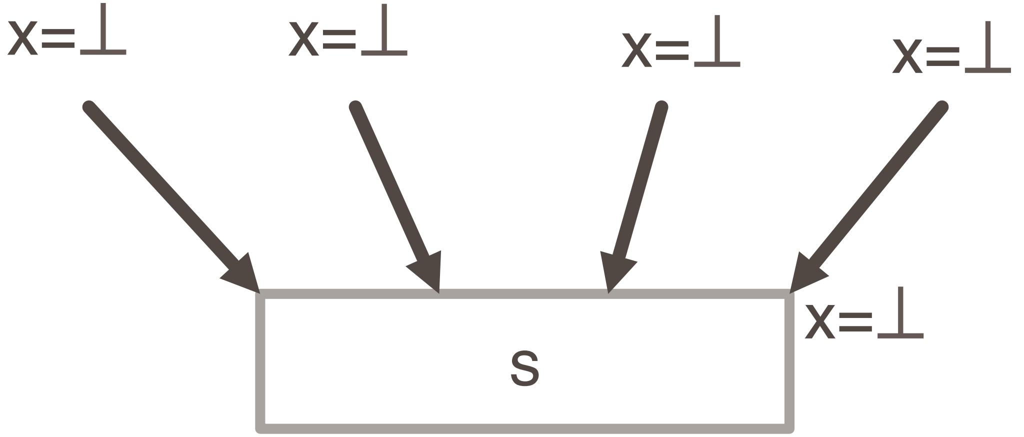
# Rule 3




if $C(p_i , x, out) = c$ or $\perp$ for all i,

then $C(s, x, in) = c$

# Rule 4

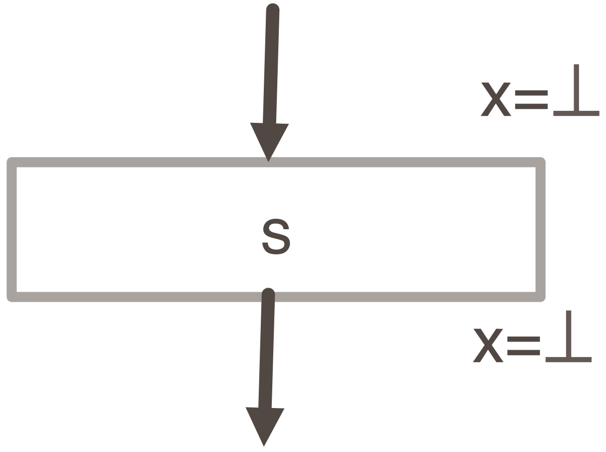



if $C(p_i\ , x, out) = \perp$ for all i,

then $C(s, x, in) = \perp$

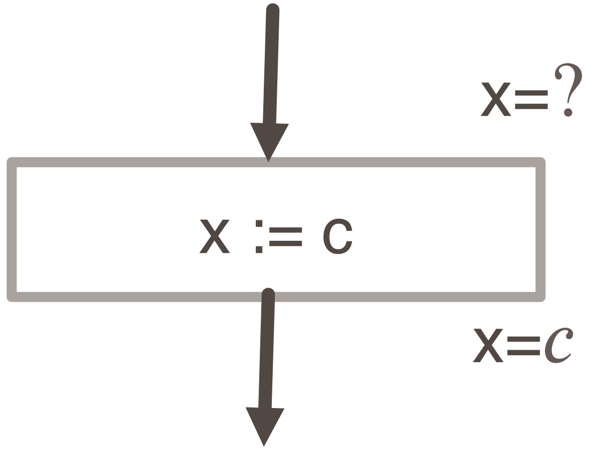
- Rules 1-4 relate the out of one statement to the in of the next statement
- Now we need rules relating the in of a statement to the out of the same statement

# Rule 5



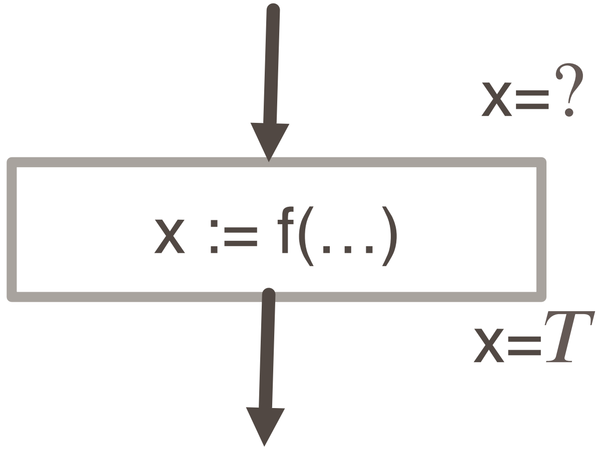

C(s, x, out) = ⊥
if C(s, x, in) = ⊥

# Rule 6



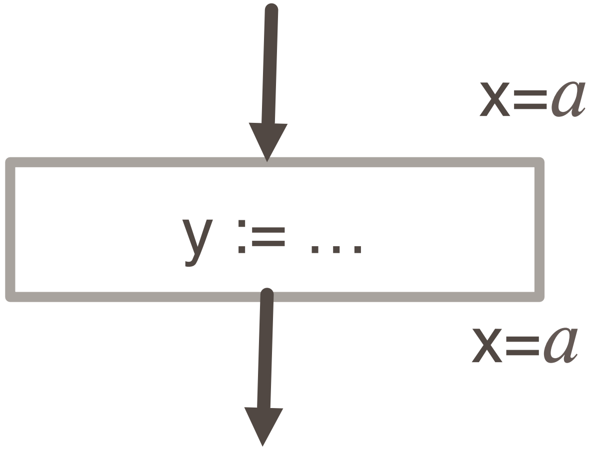

C(x := c, x, out) = c if c is a constant

# Rule 7




C(x := f(…), x, out) = T

# Rule 8




C(y := …, x, out) = C(y := …, x, in) if x <> y

# Algorithm

1. For every entry s to the program, set C(s, x, in) = $T$
2. Set C(s, x, in) = C(s, x, out) = $\perp$ everywhere else
3. Repeat until all points satisfy 1-8:
   - Pick s not satisfying 1-8 and update using the appropriate rule

- Ordering:
  - We can simplify the presentation of the analysis by ordering the values $\perp < c < T$

# Common subexpression elimination

- Example:

| | | |
|---|---|---|
| a := b + c | | a := b + c |
| c := b + c | ⇒ | c := a |
| d := b + c | | d := b + c |

- Example in array index calculations
  - c[i+1] := a[i+1] + b[i+1]
  - During address computation, i+1 should be reused
  - Not visible in high level code, but in intermediate code

# Code Elimination

- Unreachable code elimination
  - Construct the control flow graph
  - Unreachable code block will not have an incoming edge
  - After constant propagation/folding, unreachable branches can be eliminated
- Dead code elimination
  - Ineffective statements

| | | |
|---|---|---|
| x := y + 1 | | (immediately redefined, eliminate!) |
| y := 5 | ⇒ | y := 5 |
| x := 2 * z | | x := 2 * z |

  - A variable is dead if it is never used after last definition
    - Eliminate assignments to dead variables
  - Need to do data flow analysis to find dead variables

# Function Optimization

- Function inlining
  - Replace a function call with the body of the function
  - Save a lot of copying of the parameters, return address, etc.
- Function cloning
  - Create specialized code for a function for different calling parameters

# Loop Optimization

- Loop optimization
  - Consumes 90% of the execution time

    $\Rightarrow$ a larger payoff to optimize the code within a loop
- Techniques
  - Loop invariant detection and code motion
  - Induction variable elimination
  - Strength reduction in loops
  - Loop unrolling
  - Loop peeling
  - Loop fusion

# Loop Optimization

- Loop invariant detection
  - If the result of a statement or expression does not change within a loop, and it has no external side-effect
  - Computation can be moved to outside of the loop
  - Example

```
for (i=0; i<n; i++) a[i] := a[i] + x/y;
```

  - Three address code

```
for (i=0; i<n; i++) { c := x/y; a[i] := a[i] + c; }
```

⇒ c := x/y;

```
for (i=0; i<n; i++) a[i] := a[i] + c;
```

# Loop Optimization

- Code Motion
  - Reduce frequency with which computation performed
    - If it will always produce same result
    - Especially moving code out of loop

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
```

```
for (i = 0; i < n; i++) {
  int ni = n*i;
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
}
```

# Loop Optimization

- Strength reduction in loops
  - Replace costly operation with simpler one
  - Shift, add instead of multiply or divide

    ```
    16*x    -->    x << 4
    ```

    - Depends on cost of multiply or divide instruction
  - Recognize sequence of products

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
```

→

```
int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
```

# Loop Optimization

- Strength reduction in loops
  - Replace costly operation with simpler one
  - Shift, add instead of multiply or divide

    ```
    16*x   -->    x << 4
    ```

    - Depends on cost of multiply or divide instruction
  - Recognize sequence of products

```
s := 0;
for (i=0; i<n; i++)
{
  v := 4 * i;
  s := s + v;
}
```

→

```
s := 0;
for (i=0; i<n; i++)
{
   v := v + 4;
   s := s + v;
}
```

# Loop Optimization

- Induction variable elimination
  - If there are multiple induction variables in a loop, can eliminate the ones which are used only in the test condition
  - Example

    s := 0;  for (i=0; i<n; i++) { s := 4 * i; … }   -- i is not referenced in loop

    ⇒ s := 0;  e := 4*n; while (s < e) { s := s + 4; }

```
s := 0;
for (i=0; i<n; i++)
{ s := 4 * i; … }
-- i is not referenced in
loop
```

```
s := 0;
e := 4*n;
while (s < e) {
 s := s + 4;
}
```

# Code Optimization Techniques

- Loop unrolling
  - Execute loop body multiple times at each iteration
  - Get rid of the conditional branches, if possible
  - Allow optimization to cross multiple iterations of the loop
    - Especially for parallel instruction execution
  - Space time tradeoff
    - Increase in code size, reduce some instructions
- Loop peeling
  - Similar to unrolling
  - But unroll the first and/or last few iterations

# Loop Optimization

- Loop fusion
  - Example

```
for i=1 to N do
        A[i] = B[i] + 1
endfor
for i=1 to N do
        C[i] = A[i] / 2
endfor
for i=1 to N do
        D[i] = 1 / C[i+1]
endfor
```

Before Loop Fusion

```
for i=1 to N do
        A[i] = B[i] + 1
        C[i] = A[i] / 2
        D[i] = 1 /
  C[i+1]
endfor
```

# Loop Optimization

- Loop fusion
  - Example

```
for i=1 to N do
        A[i] = B[i] + 1
endfor
for i=1 to N do
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endfor
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Before Loop Fusion

```
for i=1 to N do
        A[i] = B[i] + 1
        C[i] = A[i] / 2
        D[i] = 1 /C[i+1]
endfor
```

Is this correct?
Actually, cannot fuse the third loop

# Limitations of Compiler Optimization

- Operate Under Fundamental Constraint
  - Must not cause any change in program behavior under any possible condition
  - Often prevents it from making optimizations when would only affect behavior under pathological conditions.
- Behavior that may be obvious to the programmer can  be obfuscated by languages and coding styles
  - e.g., data ranges may be more limited than variable types suggest
- Most analysis is performed only within procedures
  - whole-program analysis is too expensive in most cases
- Most analysis is based only on static information
  - compiler has difficulty anticipating run-time inputs
- When in doubt, the compiler must be conservative